Precalc Warm Up # 5-5

1. y varies directly with the cube of x and inversely with t. If y = 100 when x = 2 and t = 4, find y when x = 10 and t = 3.

Factor:

2. $8x^3 - 27$

3. $2x^3 - 3x^2 + 10x - 15$

HW Questions:

In Exercises 1–14, find a mathematical model for the verbal statement.

$\boxed{5.}$ z is proportional to the cube root of u.

$\boxed{9.}$ F varies directly as g and inversely as the square of r.

$\boxed{13.}$ *(Newton's Law of Universal Gravitation)* The gravitational attraction F between two objects of masses m_1 and m_2 is proportional to the product of the masses and inversely proportional to the square of the distance r between the objects.

$$F = \frac{k m_1 m_2}{r^2}$$

In Exercises 15–30, find a mathematical model representing the given statement. (In each case determine the constant of proportionality.)

17. A varies directly as the square of r. ($A = 9\pi$ when $r = 3$.)

21. h is inversely proportional to the third power of t. ($h = 3/16$ when $t = 4$.)

25. F is jointly proportional to r and the third power of s. ($F = 4158$ when $r = 11$ and $s = 3$.)

29. S varies directly as L and inversely as $L - S$. ($S = 4$ when $L = 6$.)

$S = \frac{kL}{L-S}$

$4 = \frac{k(6)}{6-4}$

$k = \frac{4}{3}$

$S = \frac{4}{3}\left(\frac{L}{L-S}\right)$

or $S = \frac{4L}{3(L-S)}$

don't leave it as:

$S = \frac{\frac{4}{3}L}{L-S}$

Simplify it!

Hooke's Law for a spring states that the distance a spring is stretched (or compressed) varies directly as the force on the spring.

$d = kf$

33. The coiled spring of a toy supports the weight of a child. The spring compresses a distance of 1.9 inches under the weight of a 25-pound child. The toy will not work properly if its spring is compressed more than 3 inches. What is the weight of the heaviest child who should be allowed to use the toy?

f

$1.9 = k(25)$

$k =$

In Exercises 35 and 36, use the fact that the diameter of a particle moved by a stream varies approximately as the square of the velocity of the stream.

$D = kV^2$

$0.02 = k(.25)^2$

35. A stream with a velocity of 1/4 mile per hour can move coarse sand particles of about 0.02 inch diameter. What must the velocity be to carry particles with a diameter of 0.12 inch?

39. The illumination from a light source varies inversely as the square of the distance from the light source. When the distance from a light source is doubled, how does the illumination change?

$I = \frac{k}{d^2}$

$I = \frac{k}{(2d)^2}$

$I = \frac{k}{4d^2}$

$I = \frac{1}{4}\left[\frac{k}{d^2}\right]$

The illumination is $\frac{1}{4}$ as much.

Combinations of functions

$f(x) = \sqrt{x}$ $\qquad$ $g(x) = \sqrt{4 - x^2}$

domain: $x \geq 0$ $\qquad$ domain: $-2 \leq x \leq 2$

(f+g)(x) = $\sqrt{x} + \sqrt{4-x^2}$ $\qquad$ dom? $[0,2]$

(f - g)(x) = $\sqrt{x} - \sqrt{4-x^2}$ $\qquad$ dom? $[0,2]$

(fg)(x)= $\sqrt{x}\sqrt{4-x^2} = \boxed{\sqrt{4x-x^3}}$ $\qquad$ dom? $[0,2]$

(f/g)(x)= $\dfrac{\sqrt{x}}{\sqrt{4-x^2}}$ $\qquad$ denom $\neq 0$ $\qquad$ dom? $[0,2)$

(g/f)(x)= $\dfrac{\sqrt{4-x^2}}{\sqrt{x}}$ $\qquad$ dom? $(0,2]$

Find (f + g)(1) input $\rightarrow$ is it part of the domain? yes! $[0,2]$

$\sqrt{1} + \sqrt{4-(1)^2}$

$\boxed{1+\sqrt{3}}$

When adding, subtracting, or multiplying two or more functions, the domain of the resulting function will be the intersection of the original domains.

When dividing, you have the added restriction of not being able to divide by 0.

But with a COMPOSITION, SOMETHING DIFFERENT HAPPENS!

42. The load that can be safely supported by a horizontal beam varies jointly as the width of the beam and the square of its depth and inversely as the length of the beam. Determine what happens to the safe load under the following conditions.

(a) The width and length of the beam are doubled.
(b) The width and depth of the beam are doubled.
(c) All three of the dimensions are doubled.
(d) The depth of the beam is halved.

$$L = \frac{kwd^2}{l}$$

a) load unchanged
b) load 8 times greater
c) load 4 times greater
d) load $\frac{1}{4}$ as great

c) $L = \frac{k(\cancel{2}w)(2d)^2}{\cancel{2}l}$

$$L = \frac{kw \cdot 4d^2}{l}$$

$$L = 4\left(\frac{kwd^2}{l}\right)$$

$$(f \circ g)(x) = f(g(x))$$

inside first

outcomes go into f.

Since the function g is applied first, the domain of the composite is same as the domain for g(x).

If the domain of g produces some outputs that aren't allowed to be inputs for f, then the composite doesn't exist unless we restrict the domain of g so that all of the outputs from g will work as inputs for f.

$f(x) = \frac{1}{x}$

g(x) = x + 2

Find the domain and range for f(x) and g(x).
Find f(g(x)) and state its domain and range.

dom$_f$: $x \neq 0$
range$_f$: $y \neq 0$

dom$_g$: $\mathbb{R}$
range$_g$: $\mathbb{R}$

f(g(x)) = $\frac{1}{(x+2)}$

input

The composite $f(g(x)) = \boxed{\frac{1}{x+2}}$

dom$_{f\circ g}$: = dom$_g$ $\mathbb{R}$

range$_{f\circ g}$:

check
do the outcomes of g work into f (dom$_f$)

Is this range of g (r_g) "a subset of" ($\subseteq$) the domain of f (d_f)

$r_g \overset{?}{\subseteq} d_f$

$\mathbb{R} \not\subseteq \neq 0$

so the composite doesn't exist. :(

Note: we could make it exist if we say $x \neq -2$

$f(x) = \frac{1}{x}$

g(x) = x + 2

dom$_f$: x ≠ 0

range$_f$: y ≠ 0

dom$_g$: $\mathbb{R}$

range$_g$: $\mathbb{R}$

Find the following (if it exists), and state domain and range.

g(f(x)) = $\left(\frac{1}{x}\right) + 2$

input

dom$_{f\circ g}$ = dom$_f$

$\Rightarrow \boxed{x \neq 0}$

range

$r_f \overset{?}{\subseteq} d_g$

$\neq 0 \subseteq \mathbb{R}$ yes, so

the range$_{f\circ g} \rightarrow \boxed{y \neq 2}$

f(f(x)) = $\frac{1}{\left(\frac{1}{x}\right)}$

input

Simplify: $\boxed{f(f(x)) = x}$

dom$_{f\circ f}$ = dom$_f \rightarrow \boxed{x \neq 0}$

range

$r_f \overset{?}{\subseteq} d_f$

$\neq 0 \subseteq \neq 0$

yes, so range$_{f\circ f}$ is $\boxed{y \neq 0}$

HW: PC book

p. 152 #5 - 47□, and 49

(#47c wrong answer in back of book)

* Bring SL book Monday